Application of Huygens Subgridding to Study Defibrillation in Human Body

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Introduction

- Computational Electrodynamics methods allow to study wave propagation in human body.
- Finite-Difference Time-Domain (FDTD) method is a well-suited numerical algorithm.

Motivation

- In Europe 700,000 deaths per year are accredited to a sudden cardiac arrest.
- Only effective therapy for cardiac arrest is electric defibrillation.
- Success of defibrillation depends on:
  - current level and waveform,
  - electrode size and shape, and position,
  - body size and transchordonic impedance.

Research Objective and Focus

- Enable efficient simulation of defibrillation current propagating through human body.

Frequency-Dependent-Finite-Difference Time-Domain

- Regular FDTD [1] is based on a staggered grid calculation of Maxwell's equations:
  \[
  \frac{\partial H}{\partial t} = \nabla \times E, \quad \frac{\partial E}{\partial t} = \nabla \times H, \quad \text{where}
  \]
  \[\left(\begin{array}{c}
  \frac{\partial H}{\partial t} \\
  \frac{\partial E}{\partial t}
  \end{array}\right) = \nabla \times \left(\begin{array}{c}
  E \\
  H
  \end{array}\right)
  \]
- Space and time derivatives are approximated by centred difference and average operators.
- Debye relaxation model allows to incorporate material dielectric properties:
  \[D = \epsilon e_0 \epsilon, \quad \tau = \frac{1}{\omega}, \quad \text{where} \]
- \(D\)—electric flux density vector; \(\epsilon\) and \(\epsilon\)—electric conductivity; \(\tau\)—relaxation time; \(\omega\)—angular frequency.

Huygens Subgridding Principles

- Subgridding—decomposition of simulation space into subspaces with different spatial and temporal increments \(\Delta x\) and \(\Delta t\).
- Huygens Subgridding (HSG) [2]:
  - coarse (a) and fine (b) grid influence is transferred via equivalent currents.
  - Arbitrary large subgridding ratio: \(r = \frac{\Delta x}{\Delta t}\).
  - Synchronised multistep subgridding: \(\Delta t_a = r \Delta t_b\).
  - Reduced spurious reflection from subgridding interface.
  - Drawback: late-time instability.

Figure 1: HSG principle in one dimension (1D). \(X_a\) and \(X_b\) denote coarse and fine grid regions. Simulation is performed for subspaces a, b, and c. Equivalent current from coarse to fine grid is transferred via the Inner Surface (red arrows) and from fine to coarse via the Outer Surface (blue arrows).

Propagator Environment Setting

- Three codes used for verification: HSG, FDTD (a), FDTD (b).
- Subgridding ratio \(r = 5, \Delta x_a = 5 \text{ min}, \Delta t_a = 8.95 \text{ sec}\).
- Human torso (a) 61 x 106 x 182 points (pts) and heart (b) 162 x 173 x 151 pts.
- Two defibrillator pads: anterior 84 pts and posterior 72 pts.
- Excitation source — unmodulated Gaussian pulse:
  \[f(t) = \exp\left(-\frac{t}{T}\right), \quad T = \frac{1}{f_{\text{max}}}, \quad f_{\text{max}} = 6 \times 10^3 \text{ Hz}.
  \]

Future Work

- Include Biphasic Truncated Exponential (BTE) waveform into the model.
- Conduct a comparative study of various defibrillator pads settings.

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Bibliography


Table 1: Debye relaxation media parameters, RIKEN

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