Huygens Subgridding for Auxiliary Differential Equation—
Finite Difference Time Domain
Maksims Abalënkovs¹, Fumie Costen¹, and Jean–Pierre Bérenger²
¹The University of Manchester, M60 1QD, UK, fc@cs.man.ac.uk
²Centre d’Analyse de Défense, 94114 Arcueil, France

In research areas of Microwave Imaging, Bio–Electromagnetics and –Photonics, the FDTD space frequently contains localised objects significantly smaller than other simulated structures. Subgridding techniques are applied in this type of radio environments to reduce staircasing errors and increase the size of the modelled space, at the same time effectively using the computational memory. The majority of modern subgridding schemes operate with a synchronised unistep time advancement mechanism and practically applicable subgridding ratios not exceeding 5.

Huygens Subgridding (HSG) [1] overcomes these drawbacks. The HSG design provides the opportunity for highly–independent computation of electromagnetic field in each subgrid and therefore is highly parallelisable. This paper focuses on the HSG–FDTD, expanding its application area to the aforenamed research fields by adding the Frequency Dependent (FD) material handling functionality. This paper also investigates the instability issues common to all subgridding schemes.

The work [14] in [2] applied HSG to the FD–FDTD method with \( p \)th order Debye media model which consists of
\[
- \mu_0 \frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E}, \quad \frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H},
\]
\[
\mathbf{D} = (\varepsilon_0 \varepsilon_\infty + \sum_{p=1}^{P} \frac{\varepsilon_0 \Delta \varepsilon_p}{1 + j\omega \tau_{Dp}} + \frac{\sigma}{j\omega}) \mathbf{E}
\]
where \( \Delta \varepsilon_p \) is relative permittivity change due to the \( p \)th pole and \( \tau_{Dp} \) is \( p \)th pole’s relaxation time. This scheme allows a straightforward modification and expansion of the standard FDTD equations. However, the increase of \( p \) results in higher number of equation unknowns and hence a rapid increase of memory requirements. ADE–FDTD is more lenient towards memory usage:
\[
- \mu_0 \frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E}, \quad \varepsilon_0 \varepsilon_\infty \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} - \sigma \mathbf{E} - \sum_{p=1}^{P} \mathbf{G}_p \quad \text{and} \quad \mathbf{G}_p = \frac{\partial}{\partial t} \left( \frac{\varepsilon_0 \Delta \varepsilon_p}{1 + j\omega \tau_{Dp}} \mathbf{E} \right)
\]
where \( \mathbf{G}_p \) is the polarisation current associated with \( p \)th Debye pole. Since the FD–characteristics of biological tissues tend to require \( p > 1 \), the memory–efficient ADE–FDTD has higher practical value. This paper presents a technique to integrate the HSG scheme into ADE–FDTD. It also verifies the numerical method of secondary source production and compares the results against the standard ADE–FDTD.

References
